

Definition

A sequence $w_1 \dots w_\ell$ is an n -superperm.
iff $\forall p \in S_n$ there is $i \in [\ell - n]$ s.t.

$$p(1) = X_{i+1} \dots p(n) = X_{i+n}$$

Theorem

Every n -superperm. has length $(\ell) \geq$
 $n! + (n-1)! + (n-2)! + (n-3) \dots$

Proof

Let $p, q \in S_n$, then $D(p, q)$ is the minimal number of symbols you need to append to p to get q .

$$p = 123$$

$$q = 231$$



$$\Rightarrow D(p, q) = 1$$

$$p = 123$$

$$q = 132$$



$$D(p, q) = \infty$$

$$p(1) p(2) p(n) s$$

↓
so $s = p(1)$

Therefore $\forall p \in S_n$

there is only 1 perm. $q \in S_n$
s.t. $\Phi(p, q) = 1$

$$p(1) p(2) p(3) \dots p(n) s_1 s_2$$

q

Since $p(2) \dots p(n) s_1$
is not a perm
and $s_1, s_2 \in \{p(1), p(2)\}$

$$q = p(3) \dots p(n) p(2) p(1)$$

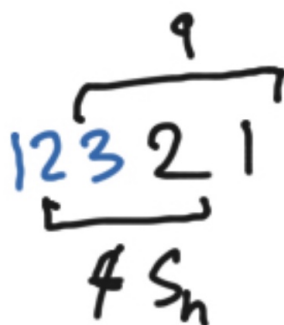
We say that $D(p, q) = k$ iff
 there are $v_1 \dots v_k$ s.t. $\begin{matrix} \cap \\ \cup \end{matrix}$
 $p(k+1) \dots p(n) v_1 \dots v_k = \sigma$

But $p(i) \dots p(n) v_1 \dots v_{i-1}$ is not a
 permut.

$p = 123$ $q = 312$ $D(p, q)$

$$p = 123$$

$$q = 321$$



$$\text{So } D(p, q) = 2$$

$$123 \rightarrow 232 \rightarrow 321$$

Let w_1, \dots, w_n be a super perm.
We know that $\{ p \in S_n : \exists i \text{ } p(i) = w_{i+1}, \dots, p(n) = w_{i+k} \}$

$$12321 \rightsquigarrow \{ 123, 321 \}$$

Let $p_1 \dots p_k$ be the perm. occur. in the sequence

For example, $12321132 \rightsquigarrow \underbrace{123}, 321, 132$
 $123123 \rightsquigarrow 123, 231, 312, 123$

Note that $n + \sum_{i=1}^{k-1} \omega(p_i, p_{i+1}) \leq L$ \uparrow
length of our sequence.

We want to prove that $d \geq n! + (n-1)$
Consider $C_0(p_1 \dots p_n) = \#$ dif. perm occur
in w_1, \dots, w_k

$$C_0(123) = 1 \quad C_0(123, 231) = 2$$

Note that $C_0(p_1 \dots p_k) = k!$ and

$$C_0(p_i) = 1$$

We claim that $D(p_1 \dots p_m) \geq C_0(p_1 \dots p_m) - 1$

$$\sum_{i=1}^{m-1} D(p_i, p_{i+1})$$

It's clear that

$$D(p_1 \dots p_m) + 1 \leq D(p_1 \dots p_{m+1})$$

and

$$C_0(p_1 \dots p_{m+1}) \leq C_0(p_1 \dots p_m) + 1$$

Note that $C_0(p_1 \dots p_k) = h!$

$$\text{So } D(p_1 \dots p_k) \geq h! - 1$$

$$\text{Therefore } l \geq h! - 1 + h = h! + (h - 1)$$