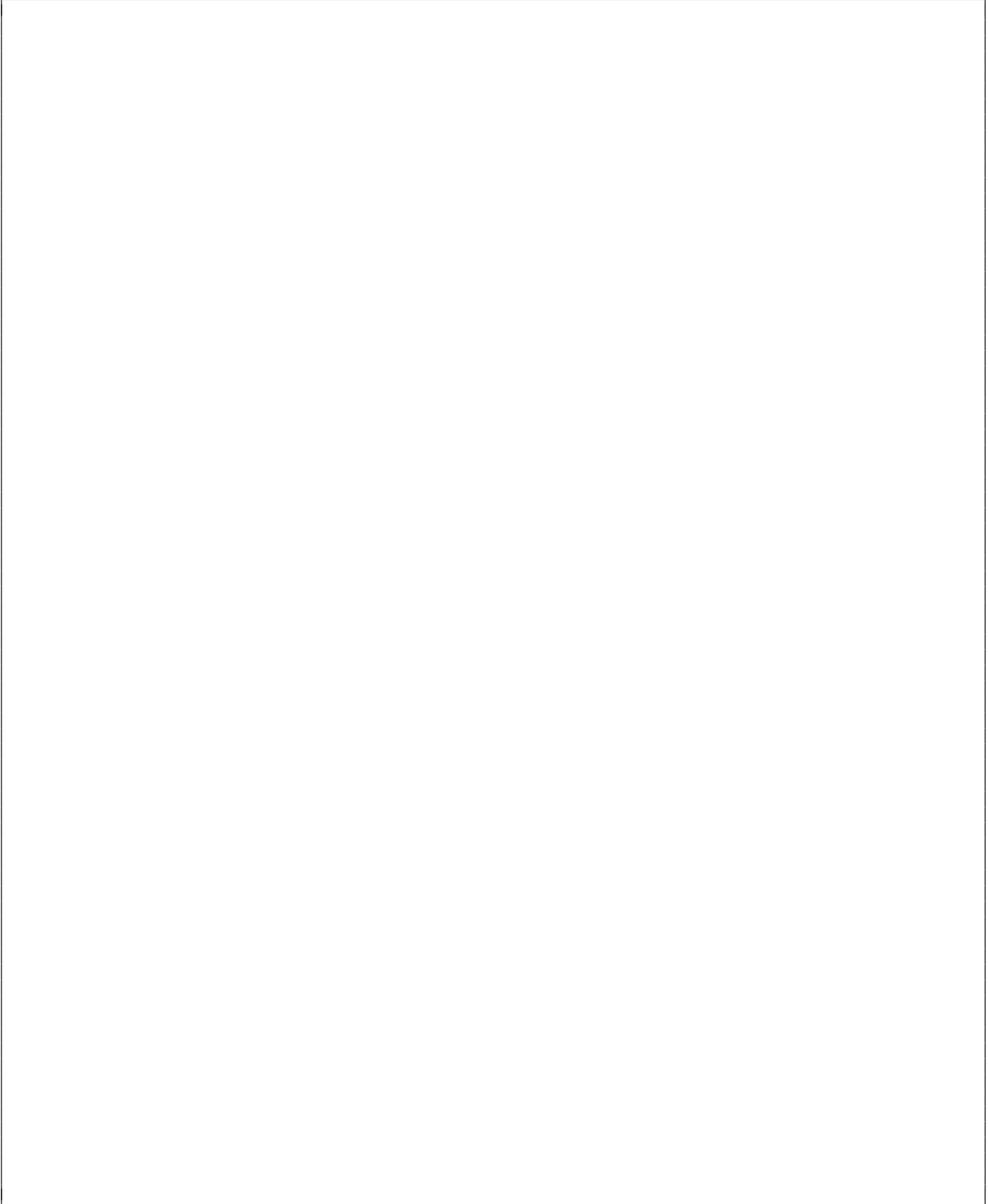


Name: \_\_\_\_\_

Pid: \_\_\_\_\_

1. Elements of  $\mathbb{Z}^2$  are colored in black and white, show that there are  $x_1, x_2, y_1, y_2 \in \mathbb{Z}$  such that  $(x_1, y_1)$ ,  $(x_1, y_2)$ ,  $(x_2, y_1)$ ,  $(x_2, y_2)$  are colored in the same color.



2. Prove the following equality:

$$\sum_{k=0}^m \binom{n+k}{k} = \binom{n+m+1}{n}.$$

3. Find a closed formula for:  $\sum_{i=0}^n i^3$ .

4. Find a recurrence relation for the number of permutations  $\pi \in S_n$  such that  $\pi^3(x) = x$  for all  $x \in [n]$ .