Name: ______________________

Pid: ______________________

1. Elements of $\mathbb{Z}^2$ are colored in black and white, show that there are $x_1, x_2, y_1, y_2 \in \mathbb{Z}$ such that (x_1, y_1), (x_1, y_2), (x_2, y_1), (x_2, y_2) are colored in the same color.

2. Prove the following equality:

$$\sum_{k=0}^{m} \binom{n+k}{k} = \binom{n+m+1}{n}.$$

3. Find a closed formula for: $\sum_{i=0}^{n} i^3$.

4. Find a reccurent relation for the number of permutations $\pi \in S_n$ such that $\pi^3(x) = x$ for all $x \in [n]$.