Name: ______________________

Pid: ______________________

1. (10 points) We say that a Boolean function $f(x_1, \ldots, x_n)$ ($f : \{0,1\}^n \to \{0,1\}$) depends on x_i iff there are $v_1, \ldots, v_{i-1}, v_{i+1}, \ldots, v_n$ such that

$$f(v_1, \ldots, v_{i-1}, 0, v_{i+1}, \ldots, v_n) \neq f(v_1, \ldots, v_{i-1}, 1, v_{i+1}, \ldots, v_n).$$

Find a closed formula (with one summation sign from 0 to n) for number of functions that depends on all their inputs.

2. (10 points) How many numbers from 1 to 1000 are neither square numbers nor cubic numbers?

3. (10 points) Let r_1 and r_2 are solutions of the equation $\lambda^2 - b_1\lambda - b_2 = 0$ and $r_1 \neq r_2$ i.e. $b_1 = r_1 + r_2$ and $b_2 = -r_1 r_2$. Find a closed formula (no summation signs) for the recurrent sequence a_n such that $a_{n+2} = b_1 a_{n+1} + b_2 a_n$ for $n \geq 0$, $a_1 = r_1 + r_2$, and $a_0 = 2$.