Lecture

Universal quantifier

$$\begin{array}{l|ll} m & A(x) & \\ & \forall y \; A(y) & \forall I, m \end{array}$$

x is a free variable in A but it's not a free variable of any open hypothesis.

$$\begin{array}{l|ll} m & \forall x \; A(x) & \\ & A(t) & \forall E, m \end{array}$$

where t is a term

Existencial quantifiers

$$
\begin{array}{r|l}
m & A(t) \\
 & \exists x\, A(x) \quad \exists I, m
\end{array}
$$

$$
\begin{array}{r|l}
m & \exists x\, A(x) \\
i & \quad \begin{array}{|l} \underline{A(y)} \\ B \end{array} \\
j & \\
 & B \qquad \exists E, m, i\text{–}j
\end{array}
$$

y isn't free in B

$\neg((\forall x\, F(x)) \vee \neg(\forall x\, F(x)))$

$\forall x\, F(x)$

$(\forall x\, F(x)) \vee \neg(\forall x\, F(x))$

$\perp$

$\neg(\forall x\, F(x))$

$(\forall x\, F(x)) \vee \neg(\forall x\, F(x))$

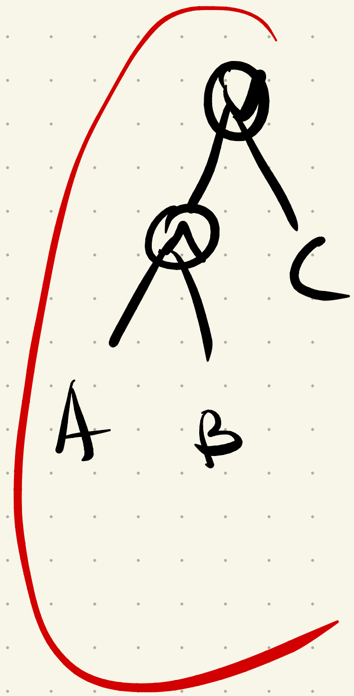
$\perp$

$(\forall x\, F(x)) \vee \neg(\forall x\, F(x))$

Derive

$$\forall x \forall y \, (R(x,y) \Rightarrow R(y,x) \wedge$$
$$R(y,x) \Rightarrow R(x,y)))$$

$(A \wedge B) \vee C$



from

$$\forall x \forall y \, (R(x,y) \Rightarrow R(y,x)))$$

1 $\forall x \forall y \; (R(x,y) \Rightarrow R(y,x))$

2 $\forall y \; (R(x,y) \Rightarrow R(y,x))$ $\forall E, 1$

3 $R(x,y) \Rightarrow R(y,x)$ $\forall E, 2$

4 $\forall y \; R(a,y) \Rightarrow R(y,a)$ $\forall E, 1$

5 $R(a,b) \Rightarrow R(b,a)$

6 $(R(x,y) \Rightarrow R(y,x)) \wedge (R(a,b) \Rightarrow R(b,a))$

7 $\forall y \; (R(x,y) \Rightarrow R(y,x)) \wedge (R(y,b) \Rightarrow R(b,y))$

8 $\forall x \; \forall y \; (R(x,y) \Rightarrow R(y,x)) \wedge (R(y,x) \Rightarrow R(x,y))$ $\forall I, 7$