Lecture 20

Let $S = (f)$ unary predicate $\mathcal{M} = (\mathbb{R}; x \geq 0)$

Consider $\varphi = (\exists x \; f(x)) \wedge f(y)$

and $s_1(v) = \begin{cases} 0 & \text{if } v = y \\ 1 & \text{otherwise} \end{cases}$

$s_2(v) = \begin{cases} 0 & \text{if } v = y \\ 2 & \text{otherwise} \end{cases}$

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$\mathcal{M} \models \varphi[s_1] \leftarrow$ it's true

$\mathcal{M} \models f(y)\,[s_1] \leftarrow$ it's true

$\mathcal{M} \models (\exists x \; f(x))\,[s_1] \leftarrow$ it's true

$\mathcal{M} \models \varphi[s_2] \leftarrow$ it's true

```
int x = 10
...
for x in range(1,11)
    print x
```

$\mathcal{M} = (\mathbb{Z}; \leq; +)$ of $\mathcal{S} = (\leq; +)$

Let $\varphi = (\exists x \; x + y \leq z)$ $\quad \mathcal{M}' = (\mathbb{N} \cup \{0\}; <, +)$

Let $s_1(v) = \begin{cases} 0 & \text{if } v = x \\ 0 & \text{if } v = y \\ 0 & \text{if } v = z \\ 0 & \text{otherwise} \end{cases}$ $\quad s_2(v) = \begin{cases} 1 & \text{if } v = x \\ 1 & \text{if } v = y \\ 0 & \text{if } v = z \\ 0 & \text{otherwise} \end{cases}$

$\mathcal{M} \models \varphi[s_1]$ ← it's true.

$\mathcal{M}' \models \varphi[s_1]$

$\mathcal{M} \models \varphi[s_1]$ ←

$\mathcal{S} = (\ddot{\smile}; \text{o\_O})$

$\varphi = (\exists x \; \ddot{\smile}(\text{o\_O}(x, y), z))$

$\mathcal{M} = (\mathbb{Z}; \leq \text{ for } \ddot{\smile}, + \text{ for o\_O})$

```
enum Var = X, Y, Z, W
int s1(v) {
    if v == X
        return 1
    else if v == y
        return 1
```

Theorem

Let S be a signature and $\mathcal{M}$ be a structure.

Consider a formula φ in this signature

then for any two assignements s_1 and s_2 s.t. s_1 and s_2 are the same for free variables of φ

$$\mathcal{M} \models \varphi[s_1] \quad \text{iff} \quad \mathcal{M} \models \varphi[s_2]$$