Lecture 15

Consider $\psi = x_1 \vee (x_2 \wedge \neg x_1)$

Let $\rho = x_1 = F, \; x_2 = T$

Note that $\psi|_\rho = T$

$$
\begin{array}{|l}
\neg x_1 \\
x_2 \\
\hline
\\
x_2 \wedge \neg x_1 \\
x_1 \vee (x_2 \wedge \neg x_1)
\end{array}
$$

We prove that if

$\psi|_{x_1=u_1 \ldots x_k=u_k} = T$, then we can derive

ψ from $x_1^{u_1} \ldots x_k^{u_k}$

if $\psi|_{x_1=u_1 \ldots x_k=u_k} = F$, the we can derive

$\neg\psi$ from $x_1^{u_1} \ldots x_k^{u_k}$

(base case) if $\psi = x_i$ for $i \in [k]$

- Consider case when $\psi|_{x_1=u_1 \ldots x_k=u_k} = T$

  So $u_i = T$, hence $x_i^{u_i} = x_i$ and

  we need to derive x_i from $\ldots x_i \ldots$

  which is easy.

- Consider case when $\psi|_{x_1=u_1 \ldots x_k=u_k} = F$

$$x_i\big|_{x_1=u_1 \dots x_k=u_k} = F \qquad \text{so} \qquad x_i^{u_i} = \neg x_i$$

$$\overset{\shortparallel}{u_i}$$

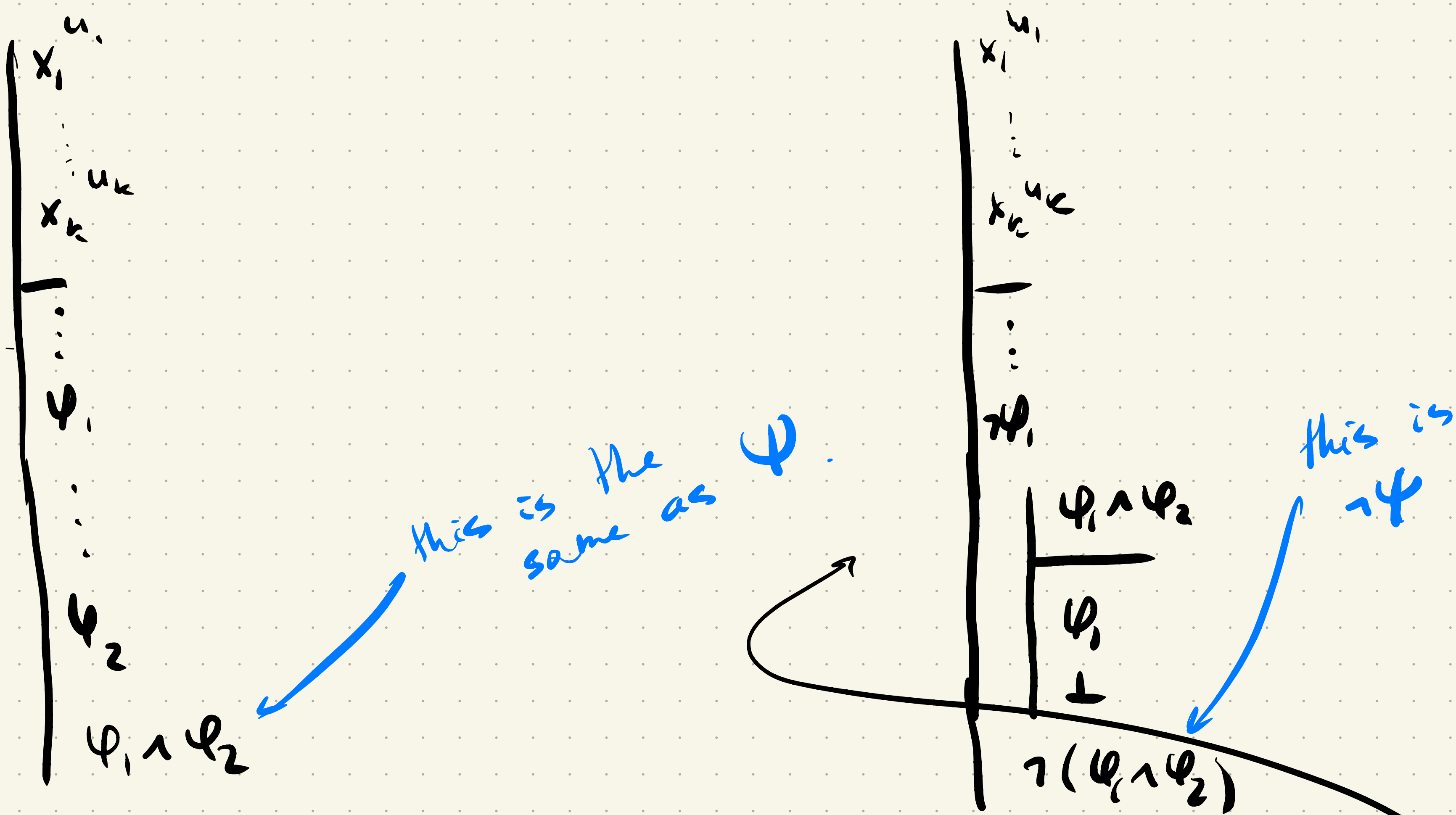
Therefore we need to derive $\overbrace{\neg\psi}^{\neg x_i}$ from

$$\underbrace{x_1^{u_1} \dots x_{i-1}^{u_{i-1}}}\ \underset{\substack{\shortparallel \\ x_i^{u_i}}}{\neg x_i}\ \underbrace{x_{i+1}^{u_{i+1}} \dots x_k^{u_k}}$$

(ind. step)

1) if $\psi = \varphi_1 \wedge \varphi_2$

- Consider case when $\psi\big|_{x_1=u_1 \dots x_k=u_k} = T$
  so $\varphi_1\big|_{x_1=u_1 \dots x_k=u_k} = \varphi_2\big|_{x_1=u_1 \dots x_k=u_k} = T$
  Hence, by IH, there is a derivation of φ_i from $x_1^{u_1} \dots x_k^{u_k}$
  for $i=1$ and $i=2$



- Consider case when $\varphi|_{x_1=u_1 \dots x_k=u_k} = F$.

WLOG $\varphi_1|_{x_1=u_1 \dots x_k=u_k} = F$

Hence, by IH, there is a der. of $\neg\varphi_1$ from $x_1^{u_1} \dots x_k^{u_k}$.