Name: ______________________

Pid: ______________________

Note that every statement in the midterm should be proved.
The only exeptions are statements that were proven in previous homework or midterms and statements proven earlier in the class.

1. (50 points) Tick if the answer for the question is **yes** (this is the only question where you do not need to prove correctness of your answer).

   ☐ Monty and a goat have apples and bananas. Monty likes apples and dislikes bananas (the more bananas he has, the worse off he is), and the goat likes bananas and dislikes apples. There are 100 apples and 100 bananas available. Is it true that the only Pareto optimal way to split fruits is to give to Monty all the apples and to the goat all the bananas?

   ☐ Is (a, b) a Nash equilibrium in the following game?

	a	b	c
a	1, 1	2, 1	5, 1
b	0, 1	0, 2	5, 0
c	4, 1	1, -1	7, 2

   ☐ Is (c, a) Pareto optimal in the following game?

	a	b	c
a	2, 1	2, 3	5, 2
b	0, 3	5, 10	1, 1
c	10, 4	2, 7	3, 4

   ☐ Ian and Masha are playing a game. In this game the starting configuration is a row of coins showing heads. The two players alternate; each player, on his or her turn, flips one coin from heads to tails and may not flip a coin next to a coin showing tails. Is 2 the value of the Grundy function of this game with 6 coins?

   ☐ Lloyd and Dunne play the following game.

	a	b
a	1, 1	2, 3
b	0, 1	4, 2

   Lloyd plays the strategy a with probability 1/3 and the strategy b with probability 2/3. Dunne plays the strategy a with probability 1/4 and the strategy b with probability 3/4.

   Is 2 the average gain of Lloyd in this case?

2. (10 points) Alice and Bob play a game with a regular polygon. Players alternate; each player, on his or her turn, draws one diagonal (a line connecting two not adjacent vertics) or draws two diagonals from the same vertex to two adjacent vertices such that none of the drawn diagonals intersect each other. A player loses when he or she has no legal move.

   (a) (10 points) Who wins in this game if the initial polygon has n vertices.

(b) (5 points) Prove that the Grundy function of this game is periodic starting from some point, i.e. there are $k, l \in \mathbb{N}$ such that $g(n+l) = g(n)$ for $n \geq k$.

You can use the fact that for $g(n+12) = g(n)$ for $300 > n \geq 100$.

3. (10 points) Eddie and Lana play a game where they each simultaneously announce an integer between 1 and 4 (inclusive). Let x be the number chosen by Eddie, and let y be the number chosen by Lana. If $x \leq y$, then Eddie wins. Otherwise, Lana wins. The losing player pays xy (i.e. the product of the two numbers) to the winning player. Construct the payoff matrix, and then find a Pareto optimal pairs of strategies (or prove that they do not exist).