Name: ______________________

Pid: ______________________

1. (100 points) Tick if the answer for the question is **yes** (this is the only question where you do not need to prove correctness of your answer).

   ☐ Is the Nim position $(3, 9, 12)$ an N-position.

   ☐ Is 8 in the subtraction game where players may subtract 1 and 4 chips on their turn an N-position?

   ☐ Is Bitwise XOR of 110011 and 010011 100000.

   ☐ Is Nim-sum of 14 and 21 27?

   ☐ Is mex$\{0, 1, 3\}$ equal to 3?

   ☐ Is (a, b) a Nash equilibrium in the following game?

	a	b	c
a	1, 1	2, 3	5, 1
b	0, 0	0, 10	0, 0
c	10, 1	1, 7	2, 2

   ☐ Is (c, c) Pareto optimal in the following game?

	a	b	c
a	1, 1	2, 3	5, 1
b	0, 0	0, 10	0, 0
c	10, 1	1, 7	2, 2

   ☐ Ian and Masha are playing a game. In this game the starting configuration is a single heap of objects, and the two players take turn splitting a single heap into two heaps of different sizes. The game ends when only heaps of size two and smaller remain, none of which can be split unequally. Is 2 the value of the Grundy function of this game for one heap with 7 objects?

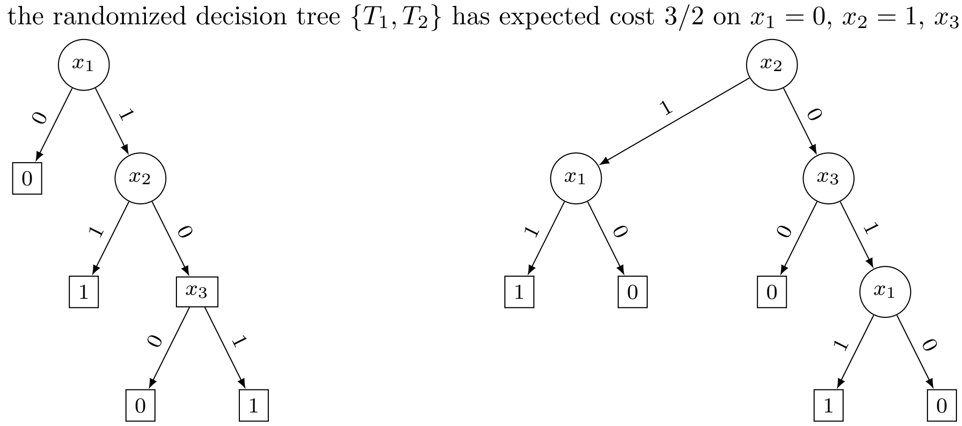
   ☐ Lloyd (the first player) and Dunne (the second player) play the following game.

	a	b
a	1, 1	2, 3
b	0, 0	0, 10

   Lloyd plays the strategy a with probability 1/2 and the strategy b with probability 1/2. Dunne plays the strategy a with probability 1/4 and the strategy b with probability 3/4.

   Is 2 the average gain of Lloyd in this case?

   ☐ Does the randomized decision tree $\{T_1, T_2\}$ has expected cost 3/2 on $x_1 = 0$, $x_2 = 1$, $x_3 = 1$?



T_1 T_2

2. (10 points) Consider the Misére subtraction game where players may subtract 2, 3 or 5 chips on their turn, identify the N and P positions.

3. (10 points) Two players one by one put kings on the 9×9 board such that none of them attack each other. Determine the winning strategy.

4. (10 points) Eddie and Lana play a game where they each simultaneously announce an integer between 1 and 4 (inclusive). Let x be the number chosen by Eddie, and let y be the number chosen by Lana. If $x + y \equiv 0 \pmod 3$, then Eddie wins. Otherwise, Lana wins. The losing player pays $|x - y|$ (i.e. the difference of the two numbers) to the winning player. Construct the payoff matrix, and find Nash equilibria in pure strategies(or prove that they do not exist).

5. Let $f : \{0,1\}^n \to \{0,1\}$ be a Boolean function such that $f(x_1, \dots, x_n) = 0$ iff $x_1 = \cdots = x_n = 0$.

   (a) (10 points) Show that $D(f) = n$.

(b) (5 points) Show that $R(f) \geq n$.

6. (5 points) We say that a randomized decision tree $\{T_i\}_{i\in I}$ represents a function f with bounded error ϵ iff for every $a \in \{0,1\}^n$, $T_i(a) = f(a)$ with probability $1-\epsilon$, i.e. for every $a \in \{0,1\}^n$ there is $J_a \subseteq I$ such that $|J_a| \geq (1-\epsilon)\cdot|I|$ and $f(a) = T_i(a)$ for every $i \in J_a$. We denote by $R_\epsilon(f)$ the minimal depth of a randomized decision tree representing f with bounded error ϵ.

   We denote by DIST_n the set of all functions $X : \{0,1\}^n \to \mathbb{R}$ (we say that X is a distribution over $\{0,1\}^n$) such that $\sum_{a\in\{0,1\}^n} X(a) = 1$.

   Finally, we say that a decision tree T heuristically represents a function f with bounded error ϵ with respect to a distribution X iff $f(a) = T(a)$ with probability $1-\epsilon$ with respect to distribution X, i.e. there is a set $C^T \subseteq \{0,1\}^n$ such that $\sum_{a\in C} X(a) \geq (1-\epsilon)$ and $f(a) = T(a)$ for every $a \in C^T$. We denote by $D_\epsilon^{(X)}(f)$ the minimal depth of a tree heuristically representing f with bounded error ϵ with respect to X.

   Show that $R_\epsilon(f) \leq \max_{X\in\mathrm{DIST}_n} D_\epsilon^X(f)$.