

Name: \_\_\_\_\_

Pid: \_\_\_\_\_

**Note that every statement in the homework should be proved.  
The only exceptions are statements that were proven in previous homework or midterms and  
statements proven earlier in the class.**

1. Alice and Bob play the following game.

- Initially, there are 20 numbers: 10 numbers 1 and 10 numbers 2.
- On each step one of the players select two numbers; and if they were the same, replace them by 2; otherwise, replace them by 1.
- Alice make the first move and they do moves one after another.

Alice wins if the last number is 1 and Bob wins if the last number is 2. Who is the winner?

**Note that this game is not a combinatorial game.**

2. In the subtraction game where players may subtract 1, 2 or 5 chips on their turn, identify the N and P positions.

3. Is the Nim position  $(1, 3, 5)$  an N-position (explain your answer)?

4. Consider the Misère subtraction game where players may subtract 1, 5 or 6 chips on their turn, identify the N and P positions.