Name: _____

Pid: _____

Show all of your work. Full credit will be given only for answers with explanations.

1. (10 points) Find the maximum and minimum values of $f(x, y) = 4x^2 + 10y^2$ on the disk $x^2 + y^2 \leq 4$.

2. (10 points) Find $\iint_R x^2 + y^2 + xydA$, where $R = [0, 1] \times [1, 2]$.

3. Consider the plane P with equation $z = 6x - 3y + 2$.

   (a) (10 points) Find the equation of a plane parallel to P and passing through the point $\langle 1, 0, -1 \rangle$.

   (b) (10 points) For which value of a is the vector $\langle -2, 1, a \rangle$ normal to the plane?

4. Let $f(x, y) = \sin(x) + \sin(y)$.

   (a) (5 points) Find the tangent planes at $\langle \pi, \pi, 0 \rangle$ and $\langle \frac{\pi}{2}, \frac{\pi}{2}, 2 \rangle$.

   (b) (5 points) Check if these planes are intersecting; if they are intersecting, find symmetric equations for the line of intersection of the planes.

5. Let $f(x, y) = 2xy$ and $g(x, y)$ be the maximum value of $D_u f(x, y)$ over all unit vectors u.
   (a) (10 points) Find the value of $g(1, 3)$.

   (b) (10 points) Find and classify all the critical points of $g(x, y)$.

6. Let $r = \langle u + v, u + v^2, u^2 + v \rangle$, where $u = \cos(x) + \cos(\pi \cdot y)$ and $v = \sin(xy)$.
   (a) (5 points) Find $\frac{\partial r}{\partial x}$ and $\frac{\partial r}{\partial y}$.

   (b) (5 points) Find the tangent plane of the surface described by the vector function r for $x = \frac{\pi}{3}$ and $y = 1$.

7. (10 points) Find the linear approximation of the function $f(x, y) = x^2 + yx$ at $\langle 1, -1 \rangle$.